Modeling of Spherical Tetrahedron Shaped Body Impact: Part 1 Impact Against Massive Plate.

Ivan Altaparmakov[1]

Abstract. **Simulations of spherical tetrahedron shaped body for ball mill are made. The impact of spherical tetrahedron shaped body against a plate was modeled via explicit Finite Element Method. Distributions of Strain and Stress and Force to time diagrams are obtained for three cases of impact. 1. Impact of spherical tetrahedron spherical surface against a plate. 2. Impact of spherical tetrahedron tip against a plate. 3. Impact of spherical tetrahedron edge against a plate. Conclusions about the workability of the spherical tetrahedron shaped body are made.**

Index Terms: **spherical tetrahedron, impact, ball mill, FEM.**
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I. Introduction

Impact of a spherical body against massive plate is a classical problem in impact mechanics [Вернер; Stronge]. The most recent researches in this area are conducted using FEM modeling of the processes [Chuan-Yu Wu, L.Y.Li and C. Thornton ; Wu C.Y., L.Y.Li and C.Thornton; E. Jaquelin, J.P.Laine, A.Bennani and M.Massenzio].

Application of impact of spherical body against massive plate with small velocity is in ball mils. Such mills are used in mining industry for ore grinding in the production of cement, clinker grinding, in thermal power for grinding coal and chemical industries for milling of commodities. In mills using balls with a spherical shape, efficiency of these aggregates is very small - 2-5%. It is important that only the grinding of ore worldwide is consumed 6% of the electricity produced so the issue of improving the efficiency of operation of ball mills is of great practical use.

One way to increase work efficiency on ball mills is the replacement of the spherical grinding bodies with such other forms. In the Republic of Bulgaria is patented grinding body in the form of spherical tetrahedron, the use of which leads to significant increases in productivity and reduction of energy consumption [L.Tzotzorkov, T.Penchev, P.Bodurov and L.Kuzev]. Because of the novelty of this form of milling body, so far there have been no theoretical studies to determine the effect of its application.

The purpose of this work is to be modeled by FEM the stress-strain state of deformation on impact of various elements of the surface of the spherical tetrahedron shaped ball on flat plate and to determine the size of the force at the time of impact.

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II. Formulation of the problem.

In the present work was studied stress-strain state and force upon impact of a spherical tetrahedron shaped ball in a solid plate, while the contact between ball and plate is with various elements of the surface of the ball - a spherical surface, edge and tip. (Figure 1.a, 1.b, 1.c).

The study was conducted as impact process is modeled by Finite Element Method (FEM). The licensed software ANSYS was used.

Corresponding governing differential equation for the problem solved by FEM is:

$$\sigma_{ij,j} + \rho f_i = \rho \ddot{x}_i \tag{1}$$

And corresponding boundary conditions: traction on boundary ∂b_1, displacement on boundary ∂b_2 and contact discontinuity along an interior boundary ∂b_3:

$$\sigma_{ij} n_i = t_i(t), \quad x_i(X_\alpha, t) = D_i(t), \quad (\sigma_{ij}^+ - \sigma_{ij}^-) n_i = 0 \tag{2}$$

Here σ_{ij} is Cauchy stress, ρ is the current density, f is the body force density, $\ddot{x}$ is acceleration, the comma denotes covariant differentiation and n_j is a unit outward normal to a boundary element of ∂b. We can write this as:

$$\int_v (\rho \ddot{x}_i - \sigma_{ij,j} - \rho f) \delta x_i dv + \int_{\partial b_1} (\sigma_{ij} n_j - t_i) \delta x_i ds + \int_{\partial b_3} (\sigma_{ij}^+ - \sigma_{ij}^-) n_j \delta x_i ds = 0 \tag{3}$$

Where δx_i satisfies all boundary conditions on ∂b_2 and the integrations are over the current geometry [LS-DYNA].

The ball has the following characteristics: radius of spherical surface 86 mm, radius of rounding is 5 mm. It is assumed that the ball is perfectly linear elastic with modulus of elasticity of steel 45 (0.45% Carbon) - 2.1E +005 MPa, i.e. not taken into account plastic deformations and strain stiffening. It is assumed that the ball falls from a height of two meters and at the moment of contact has a speed of 6.28 m / s. During the impact the plate to ensure the solidity of the slab, it has dimensions 100 x 100 x 50mm. and is made of the same material as the ball.

On Figure 1.a is shown meshing with FE of the ball and plate. The finite elements mesh is not very fine because at this stage the aim is to evaluate the approximately studied parameters. Approximation makes sense since inside the mill processes are too complex and their evaluation is statistical.

Figure 1.b shows the scheme of the bodies and the finite elements mesh for the case of a collision of the tip of the ball into the slab. Elements are relatively large because they had not pursued very high accuracy of the decision.

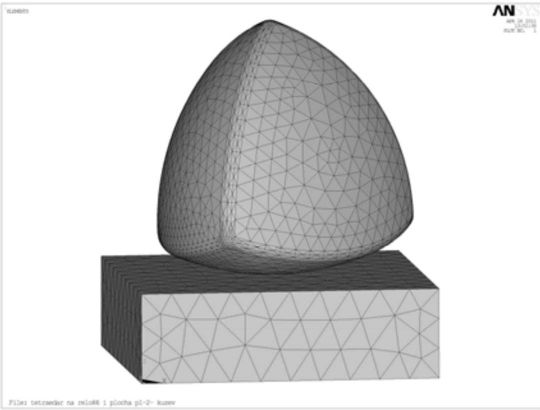

Fig. 1a. Scheme of the bodies and the finite elements mesh for the case of a collision of spherical surface of the ball in the slab.

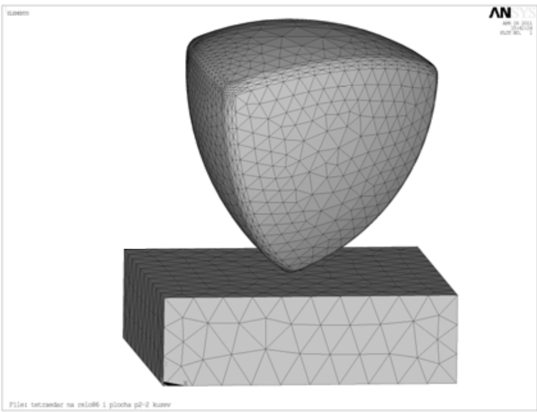

Fig. 1b. Scheme of the bodies and the finite elements mesh for the case of a collision of the tip of the ball into the slab.

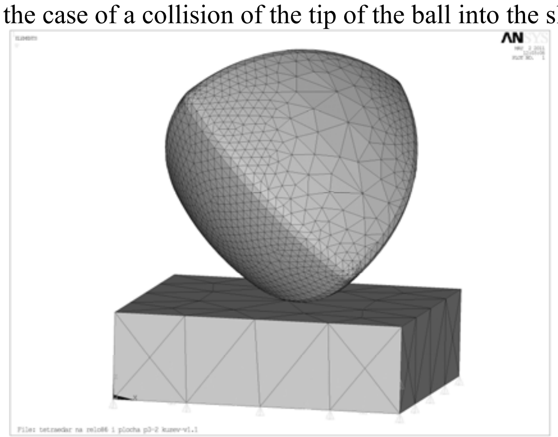

Fig. 1c. Outline of the bodies and the finite elements mesh for the case of a collision of the ball edge into the plate.

Figure 1.c shows a scheme of the bodies and finite elements mesh for the case of a collision of a ball edge into the plate. It turned out that finite elements mesh are too large, which does not allow very precise conclusions about the nature of the processes. We may need further investigation of this case with a fine finite elements mesh.

III. Results

Figures 2.a.1 and 2.a.2 shows a diagram of stress in the contact point between ball and plate and diagram of forces depending on the time of impact of the spherical surface of the ball in the slab. Contact Point is one of the heavies loaded points on impact. It is seen that stress reached almost 1125 MPa. Force reached 180 000 KN, then decreases to zero. After the impact, some fluctuations remain in stress which indicates that in the body remain to be distributed elastic shock waves or sound but with minor magnitudes. The duration of impact is about 0.2 ms.

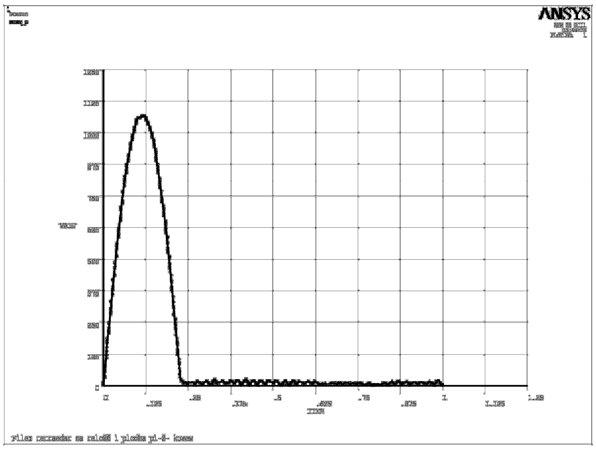

Fig. 2a.1. Diagram of stress in the contact point between ball and plate depending on the time of impact of the spherical surface of the ball in the slab

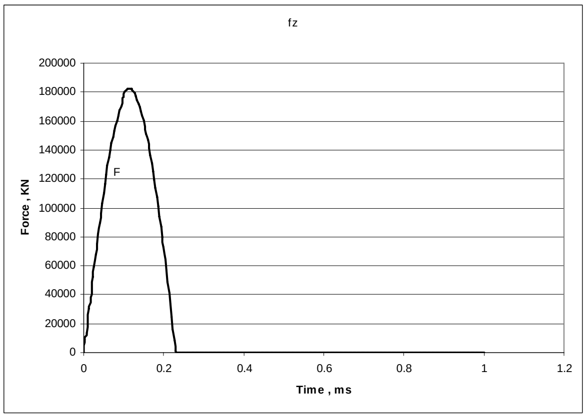

Fig. 2a.2. Diagram of forces between the ball and plate depending on the time of impact of the spherical surface of the ball in the slab.

Figure 2.b.1 and 2.b.2 shows a diagram of stress in the contact point between ball and plate and diagram of forces depending on the time of impact of the tip of the ball in the slab. In this case the stress reaches 2500 MPa - larger than in the previous case but the overall picture of the load is

similar to the previous case. Forces reach 140000KN. The duration of impact is about 0.3 ms. In this case, the residual vibrations are of small amplitude or absent at all.

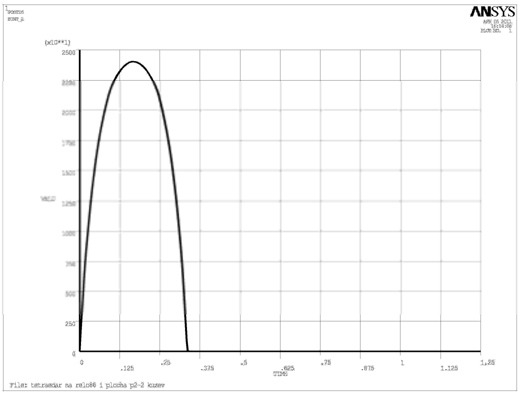

Fig. 2b.1 Chart of stress at the contact point between ball and plate depending on the time of impact of the tip of a ball in the slab.

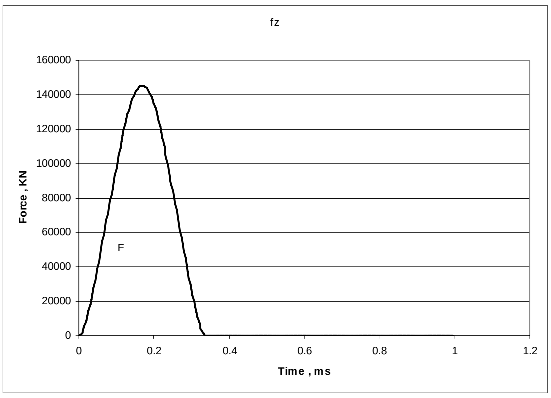

Fig. 2b.2 Diagram of forces between the ball and plate depending on the time of impact of the tip of the ball in the slab.

Figure 2.c.1 and 2.c.2 depicts the stress at the contact point between ball and plate and diagram of forces depending on the time of impact of the edge of the ball in the slab. Overall, the nature of the curve is similar to the nature of the curves of the previous cases, but in this case stress reach even greater value - to 3600 MPa. Forces reach 190 000 KN. Residual vibration in this case are not big but noticeable and larger than in the previous two cases. The duration of impact is about 0.25 ms.

Figure 3.a shows the distribution of the intensity of stress in a section which passes through the edge and through the spherical surface of the ball at the moment of the highest load. It is seen that stresses are high in the ball and reach up to 1200 MPa. Strains in the slab are almost twice as small.

Stresses in the ball are concentrated in the contact area as a zone with high stress has elongated shape toward the center of the ball. Stress on the free spherical surface of the ball are larger than the edges. Gradient of stress in the slab is smaller than the ball.

Figure 3.b shows the distribution of the intensity of stress in a section which passes through the edge and through the spherical surface of the ball at the moment of the highest load. It is seen that stresses are mainly focused at the tip of the ball and the gradient is very large. Reached value of the stress is about 41,643 MPa. Stress in the slab reaches tens of MPa.

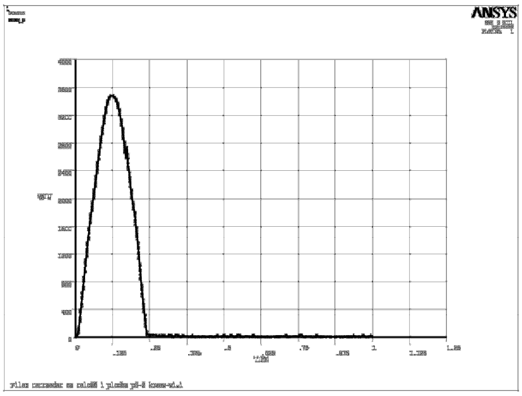

Fig. 2c.1 Diagram of stress in the contact point between ball and plate depending on the time of impact of the edge of the ball in the slab.

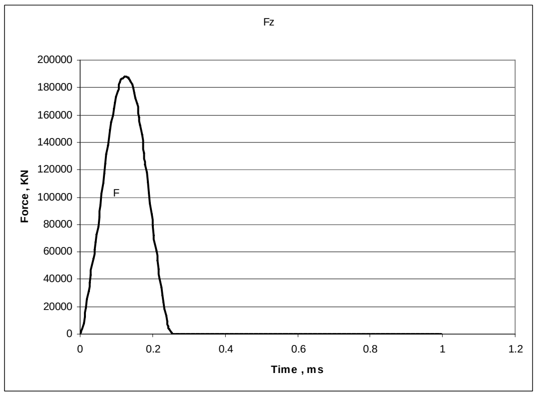

Fig. 2c.2 Diagram of forces between the ball and plate depending on the time of impact of the edge of the ball in the slab.

Figure 3.c.1 shows the distribution of the stress intensity in a section which passes through the edge perpendicular to the edge and through the spherical surface of the ball at the moment of the highest load. It is seen that stresses are mainly focused at the tip of the ball but in this case the gradient is small compared with previous cases. Interesting picture of the stresses in the slab: There is a narrow strip of high stress which penetrates inside the slab like crack - it shows that this case creates in the plate cracks and cuts it. This is very destructive effect accompanied by separation of the plate in pieces on the line parallel to the edge of the ball. Stress in the ball reaches 8000 MPa, in the plate to about 2000 MPa. The shape of the area with large stresses at the tip of the ball is typical - divided into two parts with large stress and intermediate zone with low stresses. This confirms the assertion that the impact in this case leads to separation and destruction of the plate into two parts.

Figure 3.c.2 shows the distribution of the intensity of stress in a section which passes through the edge parallel to the edge and through the spherical surface of the ball at the moment of the highest load. (Anomalous distribution of stress in the slab may be due to the fact that stresses are

concentrated in a small area that is modeled with a small number and large size of finite elements).

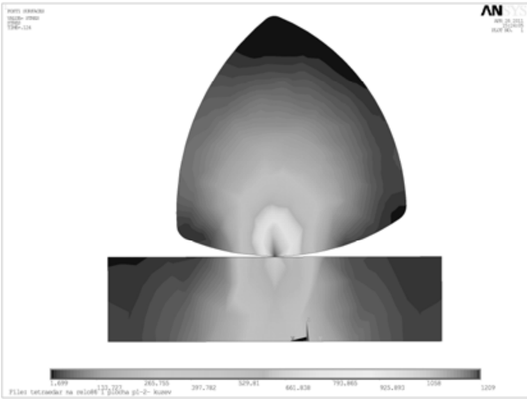

Fig. 3a. Distribution of intensity of stress in a section which passes through the edge and through the spherical surface of the ball at the moment of the highest load.

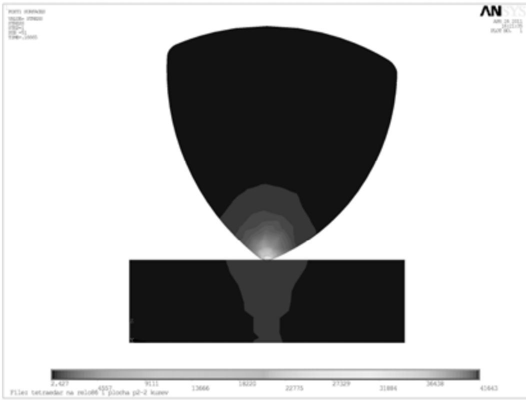

Fig. 3b. Distribution of intensity of stress in a section which passes through the edge and through the spherical surface of the ball at the moment of the highest load.

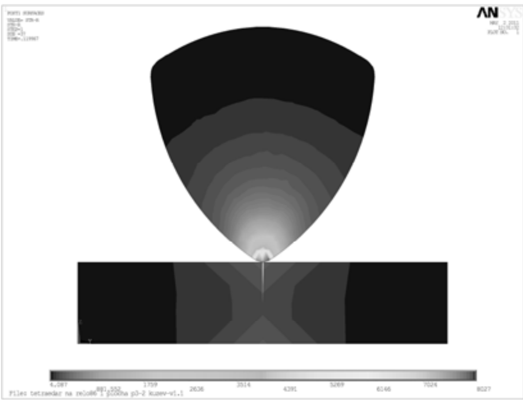

Fig. 3c.1. Distribution of stress intensity in a section which passes through the edge perpendicular to the edge and through the spherical surface of the ball at the moment of the highest load.

Stresses are concentrated again in the impact area were distributed inside the ball with a relatively low gradient. Stresses reached to 7500 MPa - less than in the section perpendicular to the edge of the ball. This can be explained by the type of area with large strains in perpendicular directions, namely two parts with large stresses side and a band of lower stresses in the middle - where the section runs parallel to the edge of the ball. In a slab there is wide strip with increased pressure - up to 6000 MPa - which again can be attributed to the separation operation on the edge of the ball. This case will require further detailed studies with a small network of finite elements.

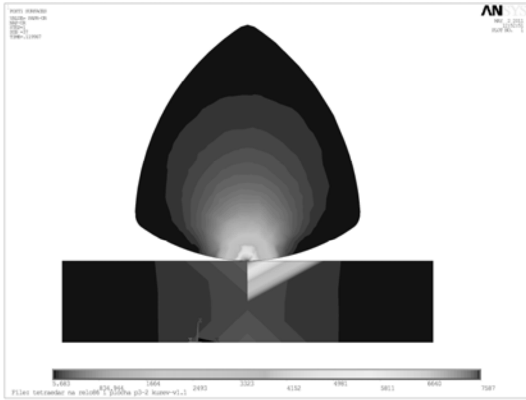

Fig. 3c.2. Distribution of stress intensity in a section which passes through the edge parallel to the edge and through the spherical surface of the ball at the moment of the highest load. (anomalous distribution of stress in the slab may be due to the fact that stresses are concentrated in a small area that is modeled with a small number and large size of finite elements)

Figure 4.a shows the distribution of the intensity of strains in a section which passes through the edge and through the spherical surface of the ball at the moment of the highest load. Again strains are larger in the ball and field with large strains is elongated form directed toward the center of the ball. In the plate strains are small and the deformed zone has a characteristic elongated shape.

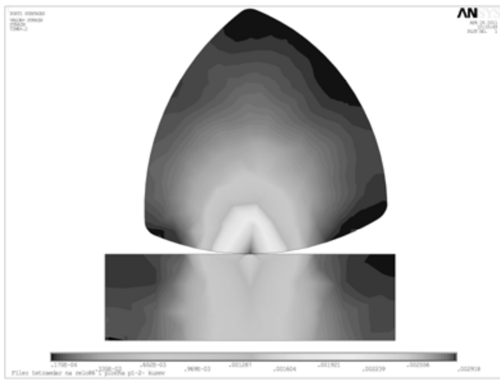

Fig.4a. Distribution of the strain intensity in a section which passes through the edge and through the spherical surface of the ball at the moment of the highest load.

Figure 4.b shows the distribution of the strain intensity in a section which passes through the edge and through the spherical surface of the ball at the moment of the highest load. Again deformations are concentrated at the tip of the ball and with very high gradient. Deformations in the slab

are concentrated in the area below the point of impact with not very large gradient.

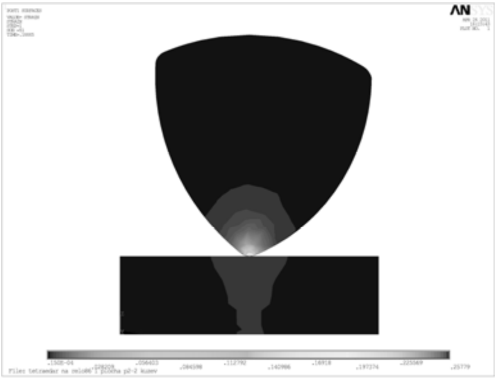

Fig.4b Distribution of the intensity of deformation in a section which passes through the edge and through the spherical surface of the ball at the moment of the highest load.

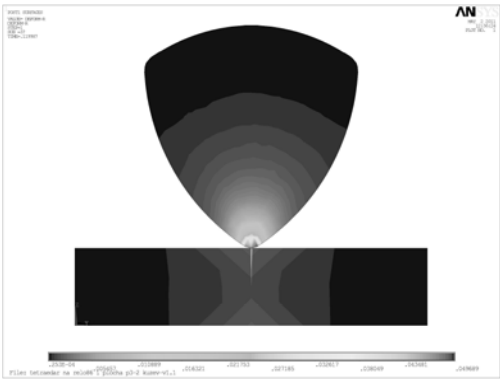

Fig.4c.1 Distribution of strain intensity in a section which passes through the edge perpendicular to the edge and through the spherical surface of the ball at the moment of the highest load.

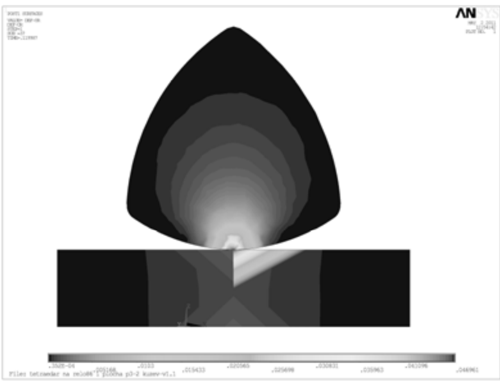

Fig.4c 2 Distribution of the strain intensity in a section which passes through the edge parallel to the edge and through the spherical surface of the ball at the moment of the highest load. (anomalous distribution of strain in the slab may be due to the fact that stresses are concentrated in a small area that is modeled with a small number and large size of finite elements)

Figure 4.c.1 and 4.c.2 shows the corresponding distribution of the strain intensity in a section which passes through the edge perpendicular to the edge and through the spherical surface of the ball at the moment of the highest load, distribution and strain intensity in a section which passes through the edge parallel to the edge and through the spherical surface of the ball at the moment of the highest load. This picture is similar to the picture of stress.

IV. Discussion of results and conclusions

As a conclusion it can be concluded that the balls are considered good by destroying power and in the edges this capability is aimed at separation of the lower bodies. Durability of the bodies is also great especially in the spherical surface due to waves which are received at the impact to the spherical surface. Waves in this case are determined by the specific shape of their bodies. Increased radius of the spherical surface lower the gradients of stresses and lowers the maximum stress, this way wear resistance is increased. Lower radius at the tips and edges increases stress gradients and maximal stress, this way the destruction capabilities of the spherical tetrahedron is increased. Significant stresses reached shows that stress stiffening at the surfaces, tips and edges will have significant impact to wear resistance of spherical tetrahedron.

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