A concept of decision-making in times of crisis and uncertainty
Maya R. Lambovska

Abstract: This paper presents opportunities for applying von Neumann – Morgenstern utility function as an instrument for choosing an alternative to overcome crisis of the organization. Choice between fuzzy triangular alternatives by means of definite (crisp) von Neumann – Morgenstern utility function is illustrated in this paper.

Index Terms: Crisis, decision-making, von Neumann - Morgenstern utility function, fuzzy triangular number.

JEL: C65, M10

I. INTRODUCTION

The aim of this paper is to present opportunities for using utility function by von Neumann – Morgenstern (hereinafter briefly called “utility function”) as an instrument for decision-making in the organization in times of crisis. In the present context decision refers to a choice of the “best” (the best possible) alternative to overcome the crisis. According to the author of this paper, type of the organization’s crisis exerts influence on the nature of alternatives to overcome it, but not the instrument for the “best” alternative choice. In this sense the author's idea could be applied to any type of organization’s crisis.

The most important limiting condition for this paper is that the environment of the organization is characterized by static, dynamic and subjective uncertainty.

The author of the paper has four purposes:

• Clarifying her view about crisis and uncertainty;
• Substantiating applicability of the utility theory of von Neumann – Morgenstern in crisis context;
• Clarifying specifics of using utility functions when alternatives to overcome the crisis are described by instruments of the fuzzy subset theory;
• Illustrating the application of utility functions in decision-making process when alternatives are represented by triangular fuzzy numbers.

II. CONCEPT OF UNCERTAINTY AND CRISIS

According to the author's viewpoint, crisis of the organization could be defined broadly as a state of extreme danger, which stems from the behaviour of the organization or of its environment factors, when this state causes exceptional difficulties in the organization functioning (Farlex, n.d.). In quantitative terms crisis finds expression in critical values of the parameters used for description of the organization’s status.

Depending on factors causing them, crises of the organization are of different nature (type) - economic, organizational, production, crisis of management and etc. (Anguelov & Stoyanov, 2013). Type of the organization’s crisis is associated primarily with the nature of:

• Critical parameters describing the organization’s status; and
• Actions to overcome the crisis.

The organization’s crisis could be overcome by taking appropriate actions by management of the organization. These actions form the content of a particular alternative of the crisis overcoming, called also “alternative solution” or “solution”. By analogy with other processes of decision making, the application of specific solution is preceded by selection from among a predefined set of alternatives.

The concept of uncertainty is crucial to the elaboration of the idea, proposed in this paper. Its author perceives uncertainty in three aspects:

• Static aspect (to a moment or moments);
• Dynamic aspect (for a time interval or intervals); and
• Subjective aspect (related to subjects' confidence).

Uncertainty in static aspect (static uncertainty) is defined as a state, characterized at any given time by lack of enough information about the number and magnitude of key environmental factors (Ostasiewicz, 1995, p. 249). Static uncertainty is explained by absence of knowledge about the object (Ramirez, 1998, p. 71).

Uncertainty in dynamic aspect (dynamic uncertainty) is defined as a change, characterized by lack of enough information about change in the number and level of uncertainty of key environmental factors during a period. The reason for dynamic uncertainty is ability of nature to change (Garcia & Perez, 2002).

Uncertainty in subjective aspect (subjective uncertainty) finds expression in vague (interval, fuzzy, probabilistic) estimates (evaluations) of the subjects. Subjective uncertainty is explained by uncertainty of subjects about their knowledge of the object (Garcia & Perez, 2002).

It is assumed in this paper that environment of the organization is uncertain in static, dynamic and subjective terms.

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III. APPLICATION OF THE UTILITY FUNCTION TO THE ORGANIZATION CRISIS

According to the author of this paper, top manager of the organization should make decisions about crises overcoming. He/she is responsible for achieving strategic goals of the organization. In many cases this process involves overcoming crisis situations. Decisions about crises overcoming are crucial for organization’s survival. Rational expectations of the top manager, including ones for the efficacy of decisions (in this context, the efficacy of alternatives to overcome the crisis), find expression in his/her utility functions by von Neumann – Morgenstern. The evaluation of alternatives of the crisis overcoming through the top manager’s utility function the provides information about what in his/her opinion is the expected usefulness for the organization of each alternative implementation, i.e. to what extent each solution is expected to overcome the crisis. Thus, by the top manager’s utility function an alternative is chosen which he/she considers is with maximum expected utility or that alternative will overcome the organization’s crisis most successfully.

Utility functions are the instrument of the theory of expected utility by von Neumann – Morgenstern. Especially, they are mathematical instrument for quantitative description of the subjective opinion of decision-making person (in this context, top manager of the organization) about alternatives potentialities to solve a problem under uncertainty (Kini & Rayfa, 1981, p. 29). In addition, utility function reflects the decision-maker’s attitude to risk (risk averse - insurance, risk seeking -speculative, and neutral). This attitude finds expression in the risk premium and its trend of change. Risk premium is calculated - as a product of the forecast (in this context, evaluation) and possibility of its occurrence (Kini & Rayfa, 1981, p. 89; Bonanno, n.d.).

Some contemporary instruments for mathematical description of qualitative evaluations (in this context, alternatives of the crisis overcoming) are the ones of the fuzzy subset theory – fuzzy subsets and fuzzy numbers. Fuzzy subset theory is a branch of mathematics. It is suitable for quantitative descriptions in terms of subjectivity and uncertainty (Kofman & Aluha, 1992, p. 11). Another case, in which the fuzzy subset theory is suitable, is for forecasting when retrospective trends should not be carried out in the future through forecasting by calculus of probability methods.

Fuzzy subset is a subset of real number set \( R \), which elements have degree of belonging (membership) to this subset in the interval \([0, 1]\). These degrees are described by a membership (characteristic) function. Fuzzy numbers (triangular and trapezoidal) are fuzzy subsets of the set of real numbers \( R \), when characteristic functions of fuzzy numbers are normal (with at least one value “unit”), convex and linear (Kofman & Aluha, 1992, p. 35). Fuzzy subset/number could by represented by so called representative number. One of these representative instruments is Hamming’s number. Hamming’s number (Kaufmann & Aluja, 1987, p. 65): a) is defined as a relative linear distance between “naught” (0) and characteristic values of the fuzzy number b) has a degree of membership (possibility of occurrence) in the interval \([0, 1]\).

In the author of this paper view, it is difficult to find mathematical solution of the problem for description of utility functions by fuzzy subsets or fuzzy numbers. The reason is the unresolved question at this stage to meet the requirement of the theory of von Neumann – Morgenstern under which sum of the possibilities (which play role of probabilities in the fuzzy subset theory) of the mutually exclusive fuzzy alternatives in a non-degenerate lottery (lottery without outcome which has probability 1) is equal to unit number (Binger, 1988, p. 497-501). In this regard, the application of both theories (the von Neumann – Morgenstern utility theory and the fuzzy subset theory) to the paper context involves a combination of alternatives, described by fuzzy subsets or fuzzy numbers, and utility functions, described by definite (crisp, with probability of realization 1) numbers.

This proposed combination of instruments requires:

- Introduction an additional limiting condition into the paper – It consists in supporting the Savage opinion about applicability of the decision-making theories, using objective probability distributions (incl. utility theory of von Neumann – Morgenstern), to subjective probability distributions (Savage, 1954);
- Using the expected utility value referred to the Hamming’s number of the fuzzy subset/ number as an evaluation instrument.

According to the author’s idea the evaluation process of alternatives to overcome the crisis covers five stages:

- Stage 1. - Determination of fuzzy alternatives to overcome the crisis;
- In the author’s view, alternatives to overcome the crisis are generated by senior management of the organization through various methods of expertise.
Mathematical description of alternatives is by fuzzy triangular numbers (see Fig. 1). Fuzzy triangular number has linear membership function and three characteristic evaluations – one evaluation for degree 1 and two evaluations for degree 0 (Kaufmann & Aluja, 1987, p. 202). This means a linear characteristic function of each alternative. That function is formed of three characteristic evaluations – one evaluation of possibility of occurrence 1 and two extreme evaluations (minimal and maximal) of possibility of occurrence 0. The value of the alternatives characteristic evaluations depends on the crisis depth.

- **Stage 2.** Determination of the decision-maker’s utility function in accordance with the criterion /criteria for the alternative choice;

  Determination of the top manager’s (decision-maker’s) utility function (see Fig. 2) regarding the alternatives efficiency is a complex process. It is divided in two sub-stages:
  - First sub-stage - interview with the top manager; and
  - Second sub-stage - mathematical determination of the top manager’s utility function.

  Top manager plays the role of interviewed person. The purpose of the interview is to identify top manager’s discrete evaluations for the utility of alternatives to overcome the crisis.

  The second sub-stage is associated with the mathematical processing of the results of the first sub-stage. This processing is realized by a variety of mathematical instruments. Kini and Raifa describe their use in detail (Kini & Raifa, 1981, p. 133, 515). One of these methods is the method of lottery of von Neumann-Morgenstern. The theory of expected utility of von Neumann-Morgenstern is based on that method. In this sense, the issues covered in the interview must be consistent with the specificity of the method of lottery.

- **Stage 3.** Determination of fuzzy evaluations of the alternatives utility;

  As a result of this stage (see Table I and Fig. 3) triangular fuzzy evaluations of the utility of various alternatives to overcome the crisis are generated. Top manager of the organization evaluates these utilities. Fuzzy utilities are determined by putting alternatives from stage 1, respectively their characteristic evaluations, into the top manager’s utility function, generated in stage 2.

- **Stage 4.** Determination of expected utilities of alternatives;

  At this stage, expected utilities are calculated for the Hamming’s representative numbers of fuzzy triangular utilities of alternatives from stage 3. This stage involves two procedures:
  - Procedure 1 - calculation of: 1) the Hamming’s number of the fuzzy utility of each alternative and 2) the degree of membership of this Hamming’s number (see columns 8 and 9 of Table I); and
  - Procedure 2 - calculation of the expected value of the Hamming’s number of the utility of each alternative (see column 10 of Table I). Expected values have a discrete nature in mathematical terms. They are shown in Fig. 3.

- **Stage 5.** Choice of a recommended alternative to overcome the crisis in the organization.

  The recommended alternative to overcome the crisis is the alternative with the maximal expected utility of the Hamming’s number. Choice at this stage is preceded by alternatives ranking according to their expected utility from stage 4 (see column 11 of Table I).

### III. APPLICATION OF THE UTILITY FUNCTION TO CRISIS SOLUTION

**The illustration** of the author’s idea about solution choice to overcome the crisis is elaborated in three variants. They are generated in connection with the different attitude to risk of the decision-maker:

- Neutral attitude to risk (function U3 in Fig. 2);
- Speculative attitude to risk (function U1 in Fig. 2);
- Insurance attitude to risk (function U2 in Fig. 2).

The present application of the utility functions meets the following limiting conditions:

- The von Neumann – Morgenstern axioms are valid (von Neumann & Morgenstern, 1947).
- Utility functions of the decision-maker are finite (limited), monotonous and continuous for each value of the criterion (von Neumann & Morgenstern, 1947).
- There is proportional aptitude (inaptitude) to risk in the decision-maker’s utility functions.
- Lotteries, used for construction of the decision-maker’s utility functions, are non-degenerate and consist of mutually exclusive alternatives (von Neumann & Morgenstern, 1947).
- Additional limiting condition from section II is valid.

Alternatives to overcome the crisis and their utilities are described by fuzzy triangular numbers (see Fig. 1).

**Results from the evaluation** process of alternatives to overcome the crisis are represented as follows:

- Result from stage 1. – in Fig. 1;
- Result from stage 2. – in Fig. 2;
- Result from stage 3. – in Fig. 3 and Table I, columns 5 to 8;
- Result from stage 4. – in Fig. 3 and Table I, columns 9 and 10;
- Result from stage 5. – in Fig. 3 and Table I, column 11.
Fig. 1. Fuzzy alternatives of the crisis overcoming in organization

Fig. 2. Variants of the decision-maker’s utility function concerning crisis overcoming

Fig. 3. Fuzzy utilities and expected utilities of alternatives of the crisis overcoming
TABLE I
RESULTS FROM THE EVALUATION PROCESS OF THE ALTERNATIVES OF CRISIS OVERCOMING

<table>
<thead>
<tr>
<th>№</th>
<th>Indicator</th>
<th>Fuzzy triangular alternative for a=</th>
<th>Fuzzy triangular utility for a=</th>
<th>Degree a of Hamming's number</th>
<th>Expected utility value</th>
<th>Alternatives ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 min</td>
<td>1</td>
<td>0 min</td>
<td>by Hamming</td>
<td>0 min</td>
</tr>
<tr>
<td>1</td>
<td>Valuation by U1 (A−)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1 Alternative № 1</td>
<td>88</td>
<td>125</td>
<td>131</td>
<td>117</td>
<td>0,4</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Alternative № 2</td>
<td>91</td>
<td>103</td>
<td>138</td>
<td>109</td>
<td>0,4</td>
<td>1</td>
</tr>
<tr>
<td>1.3 Alternative № 3</td>
<td>82</td>
<td>118</td>
<td>130</td>
<td>112</td>
<td>0,3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Valuation by U2 (A−)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1 Alternative № 1</td>
<td>88</td>
<td>125</td>
<td>131</td>
<td>117</td>
<td>0,8</td>
<td>1</td>
</tr>
<tr>
<td>2.2 Alternative № 2</td>
<td>91</td>
<td>103</td>
<td>138</td>
<td>109</td>
<td>0,8</td>
<td>1</td>
</tr>
<tr>
<td>2.3 Alternative № 3</td>
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<td>118</td>
<td>130</td>
<td>112</td>
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<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Valuation by U3 (A−)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1 Alternative № 1</td>
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<td>125</td>
<td>131</td>
<td>117</td>
<td>0,6</td>
<td>1</td>
</tr>
<tr>
<td>3.2 Alternative № 2</td>
<td>91</td>
<td>103</td>
<td>138</td>
<td>109</td>
<td>0,6</td>
<td>1</td>
</tr>
<tr>
<td>3.3 Alternative № 3</td>
<td>82</td>
<td>118</td>
<td>130</td>
<td>112</td>
<td>0,6</td>
<td>1</td>
</tr>
</tbody>
</table>

Main conclusion of the ranks in Table I (column 11) is that, under equal other conditions utility functions, reflecting different risk attitude, classify alternatives of the crisis overcoming in a different manner.

IV. CONCLUSION

This paper represents an author’s idea for choice of an alternative to overcome the crisis of the organization under uncertainty (static, dynamic and subjective).

Instruments used for the implementation of the author’s idea are utility functions by von Neumann – Morgenstern and fuzzy triangular numbers. Decision-maker’s preferences are described by utility functions and alternatives of the crisis overcoming are quantified by fuzzy triangular numbers.

REFERENCES


